DESIGN OF COMPOUND WEIRS FOR DISCHARGE MEASUREMENTS IN FLUCTUATING STREAMS

M. Akhtar Abbas, Allah Bakhsh, M. Asghar Rana & M. Amanat Ali
Department of Basic Engineering, University of Agriculture, Faisalabad

Flow measuring devices show good applicability within a limited range of discharge to be measured. However, in small watershed, when rainfall is not uniformly distributed, it ensures considerable fluctuation of flows. The existing devices are not appropriate and the compound weir is needed for flow measurement. A theory is developed whose application is reasonably justified by the laboratory experiments. The discharge equation of the form \( Q = KH^n \) is formulated. The average values of \( K \) and \( n \) are found to be 3.31 and 2.75 respectively. Such unique relationship offers a practical basis for discharge measurement. The discharge coefficients (\( C_1 \) and \( C_2 \)) for theoretically derived equation are also determined. A comparison of the theory with the observations indicates a fair agreement.

INTRODUCTION

Water is universally accepted as a national resource. The increasing use of this resource suggests the need to adopt methods for its wise and efficient use. Water flowing in open channels serves humans in many ways, and an accurate flow measurement is crucial in each of these uses. The on-site flow determination and accuracy in measurement is key to the operation of a project so that future needs may be anticipated. Weirs have been used as flow measuring devices. The compound weir composes of V-notch as lower part and upper rectangular weir. The triangular area will gauge the small flows while much larger area is available for large flows.

Webber (1971) described compound weirs as a combination of both triangular and one or more rectangular weirs. He suggested that compound weir could be calibrated utilizing the relevant formulae to determine the head-discharge relationship. White and Whitehead (1973) carried out field investigation to provide information on the calibration of compound weirs. Blaisdell and Anderson (1979) developed a mathematical model for computing the discharge over compound weir. Abbas and Wang (1989) developed and calibrated the compound weirs to justify the approach having V-notch as lower part and upper rectangular weir.

Theoretical analysis: For the purpose of analysis, the compound weir is separated into its geometrical elements as shown in Figs. 4 and 5. Using the elementary principles, the discharge equation for 90\(^\circ\) V-notch between the limits \( z = 0 \) and \( z = H \) is given by:

\[
Q_1 = \frac{8}{15} C_1 \sqrt{2gH^{5/2}} \tag{1}
\]

The flow equation for the upper rectangular part between the limits \( z = 0 \) and \( z = h \) when \( h > a \) is given by:

\[
Q_2 = \frac{4}{3} C_2 \sqrt{2gh^{3/2}} [b - 2fh] \tag{2}
\]

The resulting discharge equation is the sum of Eqs. (1) and (2).

\[
Q = \frac{8}{15} C_1 \sqrt{2gH^{5/2}} + \frac{4}{3} C_2 \sqrt{2gh^{3/2}} [b - 2fh] \tag{3}
\]

Since \( C_1 \) is not equal to \( C_2 \), the negative part can be incorporated into the triangular weir formula for simplicity:

\[
Q = \frac{8}{15} C_1 \sqrt{2gH^{5/2}} [H^{5/2} - h^{5/2}] + \frac{4}{3} C_2 \sqrt{2g} h^{3/2} \tag{4}
\]

Taking into account the end corrections, it gives:
Fig. 1. Isometric View of Experimental Setup

Fig. 2. Compound Weir-Definition Sketch

Fig. 3. Compound Weir with V-Notch Extension

Fig. 4. Analysis of Upper Part of Compound Weir

Fig. 5. Analysis of Triangular Portion of Compound Weir
\[
Q = \frac{8}{15} C_1 \sqrt{2g} \left[ H^{5/2} - h^{5/2} \right] \\
+ \frac{4}{3} C_2 \sqrt{2g} \left( b - 0.1h \right) h^{3/2} 
\]

where,
- \( b \) = horizontal base length to one side of V-notch
- \( H \) = head over V-notch crest
- \( h \) = head over horizontal crest
- \( C_1 \) = discharge coefficient for V-notch
- \( C_2 \) = discharge coefficient for rectangular section

The Eq. (5) is presented in dimensionless form to determine the discharge coefficients and is presented as:

\[
\frac{Q}{\sqrt{2g} \left[ H^{5/2} - h^{5/2} \right]} = \frac{8}{15} C_1 \\
+ \frac{4}{3} C_2 \frac{h^{3/2} \left( b - 0.1h \right)}{H^{5/2} - h^{5/2}} 
\]

The dimensionless analysis is conducted to see the relative significance of dimensionless parameters for discharge coefficients. For V-notch,

\[
f_1 \left( \rho, g, H, Q_1, \mu, \sigma, \theta \right) = 0 
\]

After doing the analysis:

\[
Q_1 = g^{1/2} H^{5/2} \frac{\rho g^{1/2} H^{3/2}}{\mu} \frac{\rho g H^2}{\sigma} \left[ \frac{1}{\theta} \right] 
\]

Comparing Eq. (8) with Eq. (1), the coefficient of discharge is:

\[
C_1 = f_1 \left[ R, W \right] 
\]

where, \( R \) is Reynolds Number and \( W \) is Weber Number. Similarly, for rectangular weir:

\[
f_2 \left( Q_2, g, h, \rho, \mu, \sigma, \theta \right) = 0 
\]

A formal dimensional analysis leads to:

\[
Q_2 = g^{1/2} h^{3/2} \frac{\rho g^{1/2} h^{3/2}}{\mu} \frac{pg^2 P}{\sigma} \left[ \frac{1}{h} \right] 
\]

Comparing Eq. (11) with Eq. (2), coefficient of discharge is:

\[
C_2 = f_2 \left[ R, W, P/h \right] 
\]

where, \( \rho \) is density, \( \mu \) is kinematic viscosity, \( \sigma \) is surface tension of water and \( P \) is crest height from the bottom of tank.

**EXPERIMENTAL INVESTIGATIONS**

Various equipment included in experiments was head tank with weir adjustment, the pointer gauges, discharge measuring unit (volumetric method), compound weirs and a diversion channel. The general sketch of the laboratory set up is shown in Fig. 1. Twelve different sizes (three sets) of sharp crested weirs were fabricated. First set had no V-notch extension into the upper part. But in second and third sets, there existed extension of width 2.9 cm and 4.8 cm respectively shown in Figs. 2 & 3. Extension was introduced to see the effect of hindrance to flow but the design dimensions were the same in all three sets. Different heads were provided and the corresponding discharge was measured volumetrically. The collected data of head and discharge were analysed to find the required results.

**RESULTS AND DISCUSSION**

Theoretically, it is difficult to predict the \( K \) and \( n \) for geometry of compound weir because the compound section composes of two distinct constituent weirs. The data recorded under the free overfall conditions were analysed using STAT. PAK software computer programme. Using power regression technique, the best average line was fitted to the observed values. The dimensional constant \( K \) and the hydraulic exponent \( n \) were found to be the intercept and the slope of the best fit line respectively. Figs. 6 & 7 exhibit the head - discharge relationship. The logarithmic plotting provides the clear definition of the slope.

The effect of extension was to reduce the flow which would occur at the same head for the same dimensions. For small flows in the lower part, triangular weir principles are observed. The average calibrated values for \( K' \) and \( n' \) are 1.31 and 2.41 respectively which show a fair agreement with the predetermined values for V-notch.

As the flow exceeds the horizontal crest,
Fig. 6. Head-Discharge Relationship for Weir of Size
a = 8.9 cm, b = 18.5 cm, and No Extension

Fig. 7. Head-Discharge Relationship for Weir of Size
a = 11.9 cm, b = 13.6 cm and Ext. Width = 2.9 cm

Fig. 8. Determination of Discharge Coefficients
for Weir of Size a = 8.9 cm, b = 18.5 cm,
and No Extension

Fig. 9. Determination of Discharge Coefficients
for Weir of Size a = 11.9 cm, b = 18.5 cm,
and Ext. Width = 2.9 cm.
compounding is applicable. For compound section, average K and n values are 3.31 and 2.75 respectively. The K values decreased with increase in "b" as shown in Table 1. The analysis revealed that n values were higher than that for triangular weir and too far away to rectangular weirs. Like K values, n values also decreased by increasing b values. The linear regression technique is applied to find C1 and C2 using Eq. (6). From Table 1, it is apparent that there are some discrepancies among the results. The reason may be approach conditions, water pulsations at higher heads, and weir dimensions. As the weir width b increases, the compounding provides more flow space that causes C1 to increase and C2 to decrease simultaneously as shown in Figs. 8 & 9. In general, it is found that large dimension weirs are better fitted to least square method than the smaller ones.

Table 1. The dimensional constant of proportionality (K), exponent of head (n) and the discharge coefficients (C1 and C2) for compound weirs

<table>
<thead>
<tr>
<th>Weir size</th>
<th>Width of V-notch extension (cm)</th>
<th>K</th>
<th>n</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (cm)</td>
<td>b (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.9</td>
<td>13.6</td>
<td>3.65</td>
<td>2.86</td>
<td>0.534</td>
<td>0.673</td>
</tr>
<tr>
<td>8.9</td>
<td>18.5</td>
<td>3.18</td>
<td>2.72</td>
<td>0.602</td>
<td>0.593</td>
</tr>
<tr>
<td>11.9</td>
<td>13.6</td>
<td>3.67</td>
<td>2.83</td>
<td>0.699</td>
<td>0.821</td>
</tr>
<tr>
<td>11.9</td>
<td>18.5</td>
<td>2.72</td>
<td>2.55</td>
<td>0.959</td>
<td>0.500</td>
</tr>
<tr>
<td>8.9</td>
<td>13.6</td>
<td>3.54</td>
<td>2.88</td>
<td>0.501</td>
<td>0.633</td>
</tr>
<tr>
<td>8.9</td>
<td>18.5</td>
<td>3.30</td>
<td>2.75</td>
<td>0.579</td>
<td>0.608</td>
</tr>
<tr>
<td>11.9</td>
<td>13.6</td>
<td>3.67</td>
<td>2.84</td>
<td>0.698</td>
<td>0.761</td>
</tr>
<tr>
<td>11.9</td>
<td>18.5</td>
<td>2.83</td>
<td>2.58</td>
<td>0.896</td>
<td>0.545</td>
</tr>
<tr>
<td>8.9</td>
<td>13.6</td>
<td>3.67</td>
<td>2.89</td>
<td>0.504</td>
<td>0.713</td>
</tr>
<tr>
<td>8.9</td>
<td>18.5</td>
<td>2.62</td>
<td>2.65</td>
<td>0.608</td>
<td>0.511</td>
</tr>
<tr>
<td>11.9</td>
<td>13.6</td>
<td>3.78</td>
<td>2.87</td>
<td>0.501</td>
<td>0.790</td>
</tr>
<tr>
<td>11.9</td>
<td>18.5</td>
<td>3.04</td>
<td>2.64</td>
<td>0.870</td>
<td>0.585</td>
</tr>
</tbody>
</table>

CONCLUSIONS

Based on the findings of the study, the conclusions are: (1) When H < a, the use of V-notch principle is recommended. When water level exceeds the horizontal crest of upper part, compounding must be used to compute the discharge. (2) The hydraulic exponent is found to vary within the range of 2.55 to 2.89. So, V-notch characteristics are dominant in such type of compound weirs. (3) The proportionality constant "K" has a range of 2.72 to 3.67. (4) The average value of C1 increases but C2 decreases within increase of the weir size.

REFERENCES


Blaisdell, F. W. and C. L. Anderson. 1979. Compound weir discharge. J. Hydraul. -
lies Division, ASAE. 105 (7): 817–826.
Webber, N. B. 1971. Fluid Mechanics for
Civil Engineers. Chapmann and Hall,
London.
effect of adverse approach conditions
on the calibration of compound weirs.
Hydraulic Research Station Report,
INT 114, Wallingford, UK.